ISI B. Math.
Physics I
Final Exam
Total Marks: 100

Answer any five questions. All questions carry equal marks.

1. Consider a charged particle of mass m and charge q entering a uniform constant magnetic field $\mathbf{B} = B_0\hat{\mathbf{k}}$. The force on the charged particle is given by

$$\mathbf{F} = \frac{q}{c}(\mathbf{v} \times \mathbf{B})$$

where c is the speed of light.

a) Show that the kinetic energy of the particle is a constant of motion. (5)

b) Given that at time $t = 0$, the particle starts from the origin with $\dot{x} = 0, \dot{y} = \dot{y}_0, \dot{z} = \dot{z}_0$, find the subsequent motion of the particle and make a rough sketch of its trajectory. How will the trajectory be affected by increasing the magnetic field? (5)

2. The potential for an isotropic harmonic oscillator is

$$V = \frac{1}{2}kr^2$$

(a) Plot the effective potential energy for the r-motion when a particle of mass m moves with this potential energy and angular momentum L about the origin. (3)

(b) By analyzing only the effective potential curve , without carrying out a solution discuss all types of motion that are possible for different initial conditions. (4)

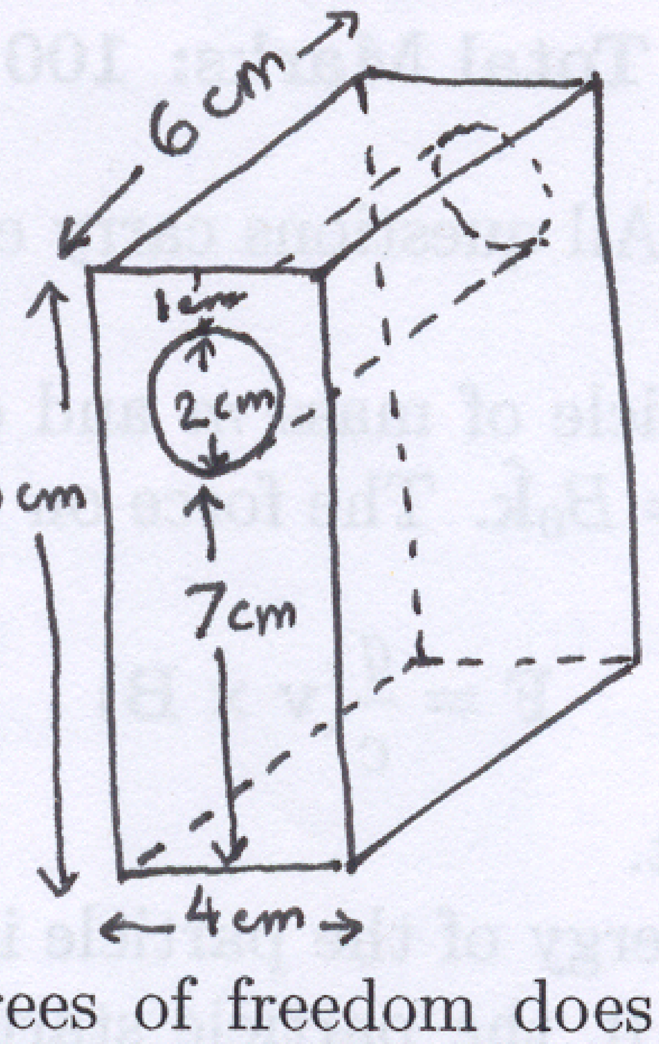
(c) Show that the actual orbit of the particle will be an ellipse for any positive total energy. Does this agree with your considerations of part (b) ? (5)

(d) Find the frequency of revolution for circular motion. (4)

(e) Find the frequency of small radial oscillations about the circular motion. (4)

3. In an experiment in which particles of mass m_1 collide elastically with stationary particles of mass m_2, it is desired to place a counter in a posiition where it will count particles which have lost half their initial momentum. At

what angle θ with the incident beam should such a counter be placed? For what range of mass ratios $\frac{m_1}{m_2}$ does this problem have an answer?



4. (a) How many degrees of freedom does an arbitrary rigid body have? Explain how you obtain the number that you state. (5)
(b) Consider a uniform rectangular block of density ρ with a cylindrical hole drilled out as shown in the figure. Find the moments of inertia of the block about axes through the centre of mass parallel to each of the three edges of the block. (15)

5. (a) Use the technique of calculus of variations to show that the curve of shortest length between two fixed points in a plane is a straight line. (10)
(b) Write down the Lagrangian $L(x, \dot{x})$ for a particle of mass m moving in one dimension under the influence of a potential $V(x)$. Show that the equations of motion obtained by minimizing the action $S = \int_{t_1}^{t_2} L dt$ (Euler-Lagrange equations) are identical to the equations of motion obtained by applying Newton's second law to this case. (5)
(c) Consider a free particle of mass m moving in one dimension with respect to a certain inertial frame K. Now consider another inertial frame K' moving with a constant velocity v with respect to K. Show that the Lagrangian L' in the frame K' differs from the Lagrangian L in K by a total time derivative of a function of coordinates and time. (5)

6. Consider a cylindrical rod made of material of rigidity modulus μ , of length L and radius a. It is rigidly clamped at the upper end and twisted by an angle ϕ by applying a torque at the other end.

(a) Show that the total torque is given by

$$\tau = c\phi$$

where the torsional rigidity $c = \frac{\mu\pi a^4}{2L}$. (10)

(b) Consider another hollow cylinder of external radius a_2 and internal radius a_1 made of the same material having the same mass and length as the solid cylinder in part (a). Let c_h and c_s be the torsional rigidities of the hollow and solid cylinders respectively. Compute c_h and show that $c_h > c_s$. That is, the torque required to twist a hollow cylinder by a specified angle is greater than that for a solid cylinder of the same mass, length and material. (10)